## Useful Formulae for ENGAA

## Physics

## Electricity

$$
R_{T}=R_{1}+R_{2}+\cdots+R_{n}
$$

Effective resistance within a series circuit

$$
\frac{1}{R_{T}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\cdots+\frac{1}{R_{n}}
$$

Effective resistance within a parallel circuit

$$
V_{\text {out }}=\frac{R_{1}}{R_{1}+R_{2}} \cdot E M F
$$

where $V_{\text {out }}$ is the PD across the resistor, $R_{1}$

$$
P=I V=\frac{V^{2}}{R}=I^{2} R
$$

where $P$ is electrical power dissipated across a component, $V$ is the PD across it and $I$ is the current through it

$$
\begin{aligned}
V & =I R \\
Q & =I t \\
W & =Q V \\
R & =\frac{\rho L}{A}
\end{aligned}
$$

where $\rho$ is resistivity, $L$ is the length of wire and $A$ is the cross-sectional area

Waves

$$
\begin{aligned}
\text { Frequency } & =\frac{1}{\text { Time Period }} \\
v & =f \lambda \\
\lambda & =\frac{a x}{D}
\end{aligned}
$$

$\lambda$ is the wavelength, $a$ is the slit to slit separation, $x$ is the fringe separation and $D$ is the slit to screen distance

$$
n \lambda=d \sin \theta
$$

$n$ is the order of the maxima, $d$ is the slit separation, $\theta$ is the angle between the light and the horizontal

## Kinematics

$$
\begin{aligned}
& \text { When acceleration is not constant: } \\
& \qquad \begin{array}{c}
a=\frac{d v}{d t} \\
v=\frac{d s}{d t}=\int a d t \\
s=\int v d t
\end{array}
\end{aligned}
$$

For a constant acceleration use suvat equations:

$$
\begin{gathered}
s=u t+\frac{1}{2} a t^{2} \\
v^{2}=u^{2}+2 a s \\
v=u+a t
\end{gathered}
$$

## Forces and Equilibrium

$$
\begin{gathered}
\text { Moment }=\text { Force } \times \text { Perpendicular Distance } \\
\qquad F \leq \mu R
\end{gathered}
$$

For an object in equilibrium, where $R$ is the normal contract force and $F$ is friction

$$
\begin{gathered}
F=\mu R \\
\text { At the point of sliding }
\end{gathered}
$$

Magnitude of Resultant Force, $\mathrm{F}=\sqrt{x^{2}+y^{2}}$

$$
\begin{aligned}
& x=F \cos \theta \\
& y=F \sin \theta
\end{aligned}
$$

where $x$ is the force in horizontal plane and $y$ is force in vertical

$$
\text { Centripetal Force }=\frac{m V^{2}}{R}
$$

Newton's Laws

$$
F=m a
$$

where F is the resultant force acting on the body
Momentum $=$ mass $\times$ velocity
where total momentum in system remains constant unless an external force acts

$$
\Delta P=F t
$$

Change in Momentum = Area under F-t graph

$$
\begin{aligned}
F & =\frac{d p}{d t} \\
\text { Power } & =\frac{\text { Energy }}{\text { Time }}
\end{aligned}
$$

## Energy

$$
\begin{aligned}
K E & =\frac{1}{2} m v^{2} \\
G P E & =m g h \\
W & =f d
\end{aligned}
$$

where $d$ is the distance travelled in direction of the force in $m$
$M E_{f}-M E_{i}=$ Work done by Driving Force- Work Done by Resistive Forces where $M E_{f}$ is the Final mechanical energy, $M E_{i}$ is the Initial mechanical energy

$$
\begin{aligned}
& \text { Mechanical Energy }=K E+G P E \\
& \text { Efficiency }=\frac{\text { useful output }}{\text { total input }} \times 100 \%
\end{aligned}
$$

## Materials

$$
\text { Density }=\frac{\text { Mass }}{\text { Volume }}
$$

$$
\text { Stress }=\frac{\text { Force }}{\text { Cross-Sectional Area }}
$$

Strain $=\frac{\text { Change in Length }}{\text { Original Length }}$

Young's Modulus $=\frac{\text { Stress }}{\text { Strain }}$

$$
\begin{gathered}
F=k x \\
E P E=\frac{1}{2} F x=\frac{1}{2} k x^{2}
\end{gathered}
$$

Area underneath a $F$-x graph

## Radioactivity

Half-life = time taken for number of undecayed nuclei or the activity of a sample to halve

$$
\text { Number of half - lives occurred, } n=\frac{\text { time elapsed }}{\text { half - life of a sample }}
$$

$$
A=\left(\frac{1}{2}\right)^{n} \cdot A_{0}
$$

The final activity, $A$, after $n$ half-lives, of a sample with initial activity $A_{0}$

$$
N=\left(\frac{1}{2}\right)^{n} \cdot N_{0}
$$

Amount of undecayed nuclei left, $N$, after $n$ half-lives, of a sample with initial number of undecayed nuclei $N_{0}$

## Mathematics

## Ratios and Proportionality

$$
\text { If } x: y=a: b \text {, then } \frac{x}{y}=\frac{a}{b}
$$

where $a$ and $b$ are numbers

$$
x \propto y \Rightarrow \frac{x_{1}}{y_{1}}=\frac{x_{2}}{y_{2}}=k
$$

$x$ is directly proportional to $y$ so their quotient is a constant ratio, $k$

$$
x \propto \frac{1}{y} \Rightarrow x_{1} y_{1}=x_{2} y_{2}=k
$$

$x$ is inversely proportional to $y$ so their product is a constant ratio, $k$

$$
\text { Given } x=y z, \text { for a constant } z, x \propto y
$$

## Algebra and Functions

For a quadratic of form $a x^{2}+b x+c=0$, roots are:

$$
\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Discriminant, $\mathrm{d}=b^{2}-4 a c$
$d=0$ for repeated roots, $d<0$ for no real roots, $d>0$ for 2 distinct real roots
If $f(a)=0$, then $(x-a)$ is a factor of the equation $f(x)=0$
If $f(a)<0, f(b)>0$, the root is in the interval $[a, b]$

## Sequences and Series

For Arithmetic Progressions with first term, $a$, and common difference, $d$ :

$$
\begin{gathered}
U_{n}=a+(n-1) d \\
S_{n}=\frac{1}{2} n[2 a+(n-1) d]
\end{gathered}
$$

For Geometric Progressions with first term, $a$, and common ratio, $r$ :

$$
\begin{gathered}
U_{n}=a r^{n-1} \\
S_{n}=\frac{a\left(1-r^{n}\right)}{1-r} \\
S_{\infty}=\frac{a}{1-r} \text { for }-1<r<1
\end{gathered}
$$

Binomial Expansions

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

## Geometry

$$
\begin{gathered}
\text { Circumference of circle }=2 \pi r \\
\text { Area of a Circle }=\pi r^{2} \\
\text { Volume of a circle }=\frac{4}{3} \pi r^{3} \\
\text { Exterior Angle }=\frac{360}{n} \\
\text { For a regular polygon }
\end{gathered}
$$

## Coordinate Geometry

Equation of a circle: $(x-a)^{2}+(y-b)^{2}=r^{2}$
For a circle with centre $(a, b)$ and radius, $r$
If two lines are perpendicular, the product of their gradients $=-1$

Trigonometry

| Angle | $0^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ | $60^{\circ}$ | $90^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\sin \vartheta$ | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos \vartheta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |
| $\tan$ Э | 0 | $\frac{\sqrt{3}}{3}$ | 1 | $\sqrt{3}$ | undefined |

From degrees to radians: multiply by $\pi$ and divide by 180
From radians to degrees: divide by $\pi$ and multiply by 180

## For $\theta$ in radians;

$$
\begin{aligned}
\text { Length of Arc } & =r \theta \\
\text { Area of a sector } & =\frac{1}{2} r^{2} \theta
\end{aligned}
$$

For a triangle with angles $A, B$ and $C$ in radians or degrees and sides $a, b, c$;

$$
\begin{gathered}
\text { Area of a Triangle }=\frac{1}{2} a b \sin C \\
\qquad \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}
\end{gathered}
$$

Trigonometric Identities;

$$
\begin{gathered}
\sin ^{2} \theta+\cos ^{2} \theta=1 \\
\tan \theta=\frac{\sin \theta}{\cos \theta}
\end{gathered}
$$

## Exponentials and Logarithms

$$
\begin{gathered}
\log a+\log b=\log a b \\
\log a-\log b=\log \frac{a}{b} \\
\log a^{b}=b \log a
\end{gathered}
$$

Calculus

> Increasing function: $f^{\prime}(x)>0$
> Decreasing function: $f^{\prime}(x)<0$
> Stationary Point: $f^{\prime}(x)=0$
> Maximum point: $f^{\prime \prime}(x)<0$
> Minimum point: $f^{\prime \prime}(x)>0$

